

SIMPLIFIED METHOD FOR DEVELOPMENT OF  
EARTHQUAKE GROUND AND FLOOR RESPONSE SPECTRA  
FOR NUCLEAR POWER PLANT DESIGN

by

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SYNOPSIS

Ground-response (or structure motion) spectra and the simplified deterministic method used for deriving them are presented. Floor-response (or equipment motion) spectra are developed directly from the ground-response spectra, using the same procedure. Analytical techniques and convenient graphical solutions are given. The results of a sample earthquake analysis, using the described method, are compared with the more tedious, conventional solution involving a real earthquake time-history. Applications relating to the seismic analysis of Canadian nuclear power plants are briefly described.

INTRODUCTION

Canadian nuclear power plants must be designed to safely withstand the maximum probable earthquake they are likely to experience during their lifetime. Unfortunately, there are very few seismographic records of Canadian earthquakes and no strong-motion accelerograms have ever been recorded which are entirely suitable for use in aseismic design of nuclear power plants in eastern Canada.

The simplicity and convenience of smoothed earthquake response-spectra for defining and establishing the aseismic requirements of nuclear power plants is well recognized. While primary nuclear structures can be adequately designed to resist an earthquake using the modal analysis-response spectrum technique, it is usually difficult to determine the expected level of response of critical control systems, reactor components and process equipment to the earthquake-induced motion of the structure on which they are mounted. It is frequently necessary to resort to the tedious, costly and often uncertain time-history method of analysis for the design of such secondary systems and equipment. This requires a suitable earthquake time-history which is compatible with the chosen design response-spectrum.

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Often, several earthquake time-histories, an artificial earthquake or a meticulously-adjusted time history have to be used, (1), so that risk of seriously underestimating or overestimating all or part of the response spectrum (and hence the structure and equipment motion) is avoided.

Similarly, power spectral density functions can be developed, using random motion or white noise, to define the expected response of the equipment to the structure (2). This approach is also tedious and produces equivalent steady-state motion with its narrow-band response, unlike the expected wide-band transient response to a real earthquake.

The method described in this paper is completely deterministic and easy to use. It permits construction of structure-motion spectra, for design of primary nuclear structures, and the use of such spectra for the direct development of equipment motion spectra for the design of attached nuclear equipment and other light-weight secondary systems.

#### DESIGN SEISMIC RESPONSE SPECTRA

Fig. 1, gives the design response spectra developed for the Canadian Nuclear Association Sub-Committee on Seismic Design which will appear in the forthcoming Canadian Code, 'Seismic Design Standard CSA-N289' (3). These spectra are based on the study of Mohraz, Hall and Newmark (MHN) which were incorporated in the NBK recommendation (4) now in general use in the U.S.A. for the design of their nuclear power plants. The MHN study showed the effect of different normalization practices and the spread in mean and mean plus one standard deviation in ground-response levels, depending on the method of normalization. Fig.1 is based on the MHN results using velocity normalization<sup>b</sup> of the mean plus one standard deviation of the 2% damped structure ground-response to 33 horizontal earthquake records, with further smoothing applied. The spectra were scaled to give 0.1g maximum ground acceleration for convenience. The result is approximately 12% higher responses than predicted by NBK, with a claimed increase in assurance of non-exceedance by any real earthquake of equal maximum ground velocity. The other values of response for structure dampings of 0 to 20% were determined using the computer program AMPFAC described below.

#### AMPFAC PROGRAM

Fig. 2 illustrates the model used in the development of response spectra. For obtaining structure response, ground motion in the form of a decaying sinusoid is applied at the base of the structure. The rate of decay is adjusted so that the structure, which is assumed to respond harmonically, builds up its motion until its peak exactly matches the response given by the chosen

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<sup>b</sup>NBK uses acceleration normalization with a proportional ground displacement that tends to exaggerate the displacement-dominant region of the response spectra.

design response spectrum<sup>c</sup> at each frequency. The rate of decay of the ground motion sinusoid<sup>d</sup> is determined graphically (Fig.6) or by an iterative procedure<sup>d</sup>.  $\beta_g$  is called the pseudo ground-damping, as it is not the true value. Only one such  $\beta_g$  need be determined for each of the parallel-line portions of the response spectra; one for the displacement (D) portion between 0.08 Hz and about 0.35 Hz, another for the velocity (V) portion between about 0.35 Hz and 3 Hz and a third for the acceleration (A) portion between about 3 Hz and 7 Hz. Below 0.08 Hz and above 7 Hz, straight lines are used to extend the spectra out to the cut-off frequencies of 0.02 Hz and 33 Hz, respectively. The other spectral lines are determined similarly by varying the structure damping  $\beta_s$  values to those desired, while utilizing the  $\beta_g$  values found previously.

The resulting harmonic motion of the structure is similarly imparted to the equipment to determine its peak response. As the equipment is considered to be very light compared to the structure, it is treated as being uncoupled (Fig.2). Again, the same three  $\beta_g$  values are used for determining the equipment response to the structure's motion, with any desired combination of  $\beta_s$  and  $\beta_e$ . As the frequency of the ground, structure and equipment are assumed to be equal for any given  $\beta_g$ , no specific frequency term is required in the equations of motion employed.

The characteristic motions of the ground, structure and equipment, for the acceleration - dominant portion of the design response-spectrum, and selected damping values, are shown in Fig. 3. The envelope of the peaks of these motions on a time scale is shown on Fig. 4. The amplification factors (AF) are merely the ratios of the peaks. For the example shown,  $AF_{\text{structure-to-ground}} = \frac{3.04}{1.0} = 3.04$ . The  $AF_{\text{equipment-to-structure}} = \frac{47.6}{3.04} = 15.7$ .

The peak values are obtained automatically by the equations of motion employed, even though the peak frequency of the responding system is usually slightly different from that of the forcing system.

#### EQUATIONS OF MOTION

Assuming that the motion of the ground, structure and equipment at any time  $t$ , corresponding to a number of  $\frac{1}{2}$  cycles or sine pulses  $n$ , are given by  $G_n$ ,  $S_n$  and  $E_n$ , respectively (see Fig.3), the equations of motion, which apply equally well to displacement, velocity or acceleration, are given as follows:

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<sup>c</sup>2% structure damping was taken; the value used in the MHN study(4).

<sup>d</sup>AECL computer program FIBGAFS

$G_n = G_1 e^{-\beta_g \omega t / \sqrt{1-\beta_g^2}} \sin \omega t$ , where  $G_1$  is always taken as unity

$e = 2.7182818$

$r_g =$  ground motion decay ratio  $\frac{G_2}{G_1} = \frac{G_3}{G_2}$ , etc =  $e^{-\beta_g \pi / \sqrt{1-\beta_g^2}}$

Similarly, when the motions of the structure and equipment are allowed to decay freely, the ratios between pulses are given as  $r_s$  and  $r_e$ , respectively, substituting  $\beta_s$  and  $\beta_e$  for  $\beta_g$ , where all  $\beta$  values are taken as decimal fractions of critical damping.

The peak steady-state response of a structure to a uniform sinusoidal ground motion of  $G \sin \omega t$  is  $\frac{G}{2\beta_s \sqrt{1-\beta_s^2}}$ , for which  $r_g = 1.00$ .

Similarly, the peak steady-state response of the equipment to uniform sinusoidal structure motion of  $S \sin \omega t$  is  $\frac{S}{2\beta_e \sqrt{1-\beta_e^2}}$ , for which  $r_s = 1.00$ .

The initial peak response  $S_1$  of the structure to the first pulse  $G_1 = 1.00$  of ground motion is  $S_1 = G_1 \frac{(1-r_s)}{2\beta_s \sqrt{1-\beta_s^2}}$

Similarly for the equipment,  $E_1 = S_1 \frac{(1-r_e)}{2\beta_e \sqrt{1-\beta_e^2}}$

The factor  $\frac{(1-r)}{2\beta \sqrt{1-\beta^2}}$  is usually greater than unity, in which case it amplifies motion, reaching a maximum of  $\pi/2$  for zero damping and a minimum of about 0.943 for 60% damping. Thus, when  $\beta_s = 0$ , the peak steady-state motion achieved in the structure is  $\frac{\pi/2}{(1-r_g)}$ .

Each sine pulse of ground motion will normally impart amplified motion to the structure. Between pulses, the structure motion decays. Peak motion occurs in the structure when the amplified motion from a given ground pulse just equals the decay of structure motion before the next pulse arrives.

$S_1 = \frac{(1-r_s)}{2\beta_s \sqrt{1-\beta_s^2}}$  the amplified structure response per unit pulse of ground motion =  $S$

$S_2 = S_1 r_s + S r_g$  = decayed first pulse + second pulse from decayed ground motion

$S_3 = S_2 r_s + S r_g^2$

$S_n = S_{n-1} r_s + S r_g^{n-1}$

$S$  peak = maximum value of  $S_n$ , after which structure motion decays away (Fig.4).

Similarly, for equipment response to structure motion

$$E = \frac{(1-r_e)}{2\beta_s\sqrt{1-\beta_s^2}} = \text{the amplified equipment response per unit pulse of structure motion}$$

$$E_1 = S_1 E$$

$$E_2 = E_1 r_e + S_2 E$$

$$E_3 = E_2 r_e + S_3 E$$

$$E_n = E_{n-1} r_e + S_n E$$

$E_{\text{peak}}$  = maximum value of  $E_n$ , after which equipment motion decays away (Fig.4).

#### AMPLIFICATION FACTORS

A plot of equipment-to-structure amplification factors,  $AFe/s$ , over a range of structure dampings and for two selected equipment dampings is shown in Fig. 5. This also shows the relatively small effect of pseudo-ground damping  $\beta_g$ , especially at light structure damping.

Fig. 6 is a plot of structure-to-ground amplification factors,  $AFs/g$ , from which a  $\beta_g$  value can be found directly which gives the same  $AFs/g$  as that of a given ground response spectrum. Having this  $\beta_g$  allows other  $AFs/g$  values to be found for a range of structure dampings  $\beta_s$ . The cut-off line shown is the point beyond which any  $AFs/g$  becomes a constant for increasing  $\beta_g$ . The curves should not be used beyond this cut-off line to avoid nonlinearities.

Fig. 7 is generated by plotting  $AFs/g$  against  $\beta_g$  for various values of  $\beta_s$ . By changing the ordinate to  $AFe/s$  and the curves to read values of  $\beta_e$ ,  $AFe/s$  can be obtained directly for any combination of  $\beta_s$  and  $\beta_e$  in the manner shown by the example. The value of  $AFe/s$  for  $\beta_e = 0$  is calculated as follows, for the example given:

Assuming  $\beta_s = 5\%$  and  $\beta_g = 9.13\%$  (acceleration region of design response spectra, Fig. 1).

$$AFe/s (\beta_e = 0\%) = \frac{AFs/g (\beta_s = 0\%)}{AFs/g (\beta_s = 5\%)} \times AFs/g (\beta_g = 0\%)$$

$$\text{From Fig. 1 } AFs/g (\beta_s = 0\%) = 6.28$$

$$AFs/g (\beta_s = 5\%) = 3.04$$

$$AFs/g (\beta_g = 0\%) = \frac{1}{2\beta_s\sqrt{1-\beta_s^2}} = 10.0125$$

$$\therefore AFe/s (\beta_e = 0\%) = \frac{6.28}{3.04} \times 10.0 = 20.7$$

This value is located on the  $\beta_e = 0\%$  curve and joined by a straight line to  $A_{Fe/s} = \pi/2$  which is the maximum  $A_{Fe/s}$  for  $\beta_s = 100\%$  and  $\beta_e = 0\%$  (inferred from Fig. 6). The required values of  $A_{Fe/s}$  are located at intersections of this line and the curves for any chosen value of  $\beta_e$ . The error is only  $\pm 5\%$  over the full range of values of  $\beta_e$  indicated and for the same range of  $\beta_s$ .

#### ENVELOPE RESPONSE SPECTRA

A set of envelope response-spectra are given in Fig. 8 for the assumed dampings indicated. The ground motion and two structure-motion spectra are taken from Fig. 1, all plotted against acceleration.

The equipment response to the structure used  $A_{Fe/s}$  values calculated by AMPFAC for the  $\beta_g$  values shown on Fig. 5, covering the three regions indicated. One additional value of  $A_{Fe/s}$  was calculated for a frequency of 15 Hz, for which the required  $\beta_g$  was found to be 31.43%. The resulting equipment response to the structure is shown extrapolated out beyond 15 Hz to 100 Hz, where its motion is still well above the ground level. As the design response-spectra define a cut-off frequency of 33 Hz, where amplified response of the structure ceases, it is claimed that equipment response to the structure must also become unity. This is catered for by the construction shown in Fig. 9, where a normal-log plot allows a reasonable roll-off tangent line to be constructed down to the cut-off frequency of 33 Hz. As Fig. 8 gives a spectrum of equipment amplified motion for all possible combinations of ground, structure and equipment frequencies, the default value of the structure peak acceleration = 0.304g is taken to apply at and beyond 30 Hz, as this is the acceleration that high-frequency (rigid) equipment will see if resting on a structure having a natural frequency within the range 2.8 to 7 Hz. This construction is shown in the solid line of equipment response to the structure on Fig. 8 (log-log plot).

#### AMPLIFIED MOTION SPECTRA

Fig. 10 shows the response of a 5% damped structure, with a natural resonant frequency  $f_s$  of 4 Hz, to ground motion in the frequency range  $f_g = 0.4$  to 40 Hz. The structure response follows the curve  $As_4$ , where  $A's_4$  (dotted) is the curve that would result if the maximum ground motion remained at 0.100g down to 0.4 Hz or lower. On the low-frequency side of  $f_s$  (left-hand side), the structure response approaches ground motion, which it theoretically reaches at 0 Hz. On the high-frequency side of  $f_s$  (right-hand side), the structure response goes to zero as the frequency increases to  $\infty$ .

Curve  $As_4$  is a transmissibility curve determined using an adaptation of the program SPECEQ (5). This program digitizes the decaying ground motion sine wave into a series of closely-spaced acceleration impulses. A numerical integration of the resulting structure motion is made using a modified form of the Duhamel convolution or superposition integral. The resultant peak occurs just below 4 Hz and agrees very closely with the AMPFAC result

(AMPFAC always gives slightly higher peak responses).

Curve  $As_4$  lies between the steady-state response and the transient response value, which can be readily determined with sufficient accuracy as follows:

Let  $A's_4$  minimum = 0.100g  $\therefore As_4$  (at any f) = 0.1 AFs/g

and  $AFs/g = AF_{ss} + C_s (AF_{tr} - AF_{ss})$

Where  $AF_{ss} = \frac{1}{\sqrt{(1-R^2)^2 + (2\beta_{ps}R)^2}}$  steady state

$AF_{tr} = \frac{1}{\sqrt{(1-R)^2 + (2\beta_{ps}R)^2}}$  transient

$C_s$  = interpolation coefficient (Fig.11)

$R$  = frequency ratio  $f_g/f_s$

$f_g$  = frequency of ground motion

$f_s$  = frequency of structure = 4 Hz

$\beta_{ps}$  = a pseudo structure damping (decimal), chosen so that  $2\beta_{ps} \approx \frac{1}{AFs/g(\text{peak})}$

$AFs/g(\text{peak}) = 3.04$  from the spectrum, Fig.1.

The final value of  $\beta_{ps} = 17.1\%$ , as against  $\beta_s = 5\%$ . The higher pseudo-damping value is because of the transient nature of the structure motion.

The correct value of  $\beta_{ps}$  must be found by interpolation, as the peak value will occur at a frequency ratio  $R$  less than 1.00 (in this case  $R_{\text{peak}} = 0.95$ , so that  $f_s \text{ peak} = 3.8$  Hz)

$$R_{\text{peak}} (\text{steady state}) = \sqrt{1 - 2\beta_{ps}^2}$$

$$R_{\text{peak}} (\text{transient}) = \frac{1}{1 + 4\beta_{ps}^2}$$

The actual  $R$  peak will lie between these two ratios.

The left-hand value of  $As_4 = \frac{A's_4}{A_g(\text{max})} \times A_g(\text{at } f_g)$

Where  $A_g(\text{at } f_g)$  = ground acceleration at ground frequency  $f_g$

$A_g(\text{max})$  = maximum ground acceleration = 0.1g

Straight-line envelopes with acceleration maxima at 0.100, 0.304, 0.531 and 4.76g are taken from Fig. 8. They are respectively, the maximum ground motion, maximum 5% damped structure motion, 0.5% damped structure motion and 0.5% damped equipment motion (where the equipment is mounted on the 5% damped structure).

The equipment response to the 5% damped structure, with  $f_s = 4$  Hz, is given by curves  $A_{eg}$  and  $A_{es}$ .  $A_{es}$  is found using SPECEQ for a range of equipment frequencies  $f_e$  between 0.4 and 40 Hz for motion imparted by the structure at its resonant frequency  $f_s = 4$  Hz. At very high equipment frequencies, the equipment response falls to the maximum structure acceleration  $A_{s_s} = 0.304g$ .<sup>e</sup> At very low frequencies the equipment response  $A'_{es}$  falls toward zero.

$A'_{eg}$  is found similarly, where  $f_e$  and  $f_g$  are kept equal and varied together between 0.4 and 40 Hz, while the structure resonant frequency  $f_s$  remains at 4 Hz. At very low equipment frequencies, the equipment response falls to the maximum acceleration it would see if it were resting directly on the ground<sup>f</sup>. The final curve  $A_{eg} = \frac{A'_{eg}}{A_{s_e}(\text{peak})} \times A_{s_e}(\text{at } f_e)$

Where  $A_{s_e}(\text{peak}) = \text{peak structure acceleration} = 0.531$  with  $\beta_s = \beta_e = 0.5\%$ , and  $A_{s_e}(\text{at } f_e) = \text{structure acceleration at equipment frequency } f_e$ .

It is important to note that  $A'_{eg}$  can be derived directly from  $A'_{es}$  with little error (always conservative) as follows:

$$A'_{eg} \simeq A'_{es} + A_{s_e}(\text{peak}) = A'_{es} + 0.531$$

This is good to  $f_e/f_s = 0.9$ , above which  $A'_{eg} \simeq A'_{es}$

The maximum error found for the example given was about + 9% at  $f_e = 3.33$  Hz ( $f_e/f_s = 0.83$ ). For higher  $\beta_s$  or  $\beta_e$  values, where  $A_{Fe/s}$  is lower,  $A'_{eg}$  would be even more conservative. This is usually acceptable for off-peak response and may even be desirable for an extra margin of confidence.

As for curve  $A_{s_4}$ , curves  $A'_{es}$  and  $A_{es}$  can be found by interpolating between  $A_{Fss}$  and  $A_{Ftr}$ , using the interpolation coefficient  $C_e$  (Fig.11), with  $\beta_{pe}$  chosen in a similar manner to obtain  $A_{Fe/s}(\text{peak}) = \frac{A_e(\text{peak})}{A_{s_s}(\text{peak})} = \frac{4.76}{0.304} = 15.7$ .

and  $R = f_s/f_e$ . In this case  $\beta_{pe} = 3.19\%$ , against  $\beta_e = 0.5\%$ .

The peak response occurs at almost exactly  $f_s/f_e = 1.00$ . In practice, some broadening of the peak is recommended to cater for errors in calculating structure or equipment frequencies, nonlinearities, etc.

<sup>e</sup>At high frequencies the equipment acts as a rigid extension of the structure and picks up structure motion with no relative movement.

<sup>f</sup>At low frequencies, the structure acts as a rigid extension of the ground and the equipment amplifies its motion as if it were resting directly on the ground.



A family of equipment amplification (transmissibility) curves can be constructed for a range of peak values for convenience in performing computations.

#### EQUIPMENT MOTION SPECTRA

While Fig. 10 is for one structure mode and single degree-of-freedom systems, equipment motion or floor-response spectra can be derived for any number of modes of the structure. Referring again to Fig. 10 and the above discussion, the following can be used:

For each structure mode  $n$  and at a particular equipment frequency  $f_e$

$$\begin{aligned} Ae(\text{right})_n &= (As_n \cdot \Gamma_{sn} \cdot \phi_{sn} \cdot A_{Fe}/s) \\ &= (Aes \cdot \Gamma_{sn} \cdot \phi_{sn}) \end{aligned}$$

Where:  $As_n$  = Acceleration of structure for frequency of mode  $n$  from design response-spectra at damping  $\beta_s$ .

$\Gamma_{sn}$  = Participation factor of structure for mode  $n$

$\phi_{sn}$  = Shape factor (eigenvector) for structure in mode  $n$  at point of equipment support.

$A_{Fe}/s$  = Amplification factor at  $f_e$  of equipment-to-structure =  $\frac{Aes \text{ (at } f_e)}{As_s \text{ (peak)}}$

$$Ae(\text{left})_n = \left[ \frac{A'es \cdot \Gamma_{sn} \cdot \phi_{sn}}{As_e \text{ (peak)}} + 1 \right] As_e \text{ (at } f_e)$$

This expression would apply if there were only one structure mode  $n$ . For several modes, only part of this expression is computed, before summing the various modal responses as follows:

$$A'e(\text{left})_n = A'es \cdot \Gamma_{sn} \cdot \phi_{sn}$$

The combined response  $Ae(\text{total})$  is based on the square root of the sum of the squares (RSS) of the separate modal response values for the chosen equipment frequency.

$$Ae(\text{right})^2 = \sum_{n=1}^N (Aes \cdot \Gamma_{sn} \cdot \phi_{sn})^2$$

$$Ae(\text{left})^2 = \left[ \left( \frac{\sqrt{\sum_{n=1}^N (A'es \cdot \Gamma_{sn} \cdot \phi_{sn})^2}}{As_e \text{ (peak)}} + 1 \right) As_e \text{ (at } f_e) \right]^2$$

$$Ae(\text{total}) = \sqrt{Ae(\text{right})^2 + Ae(\text{left})^2}$$

The above is repeated for as many equipment frequencies as desired, including those which coincide with the structure modal frequencies, until the complete equipment motion spectrum is obtained. The same procedure is followed for each level (floor) of the structure at which equipment is to be supported.

#### COMPARISONS

The maximum structure-to-ground acceleration amplification factor, taken from NBK (4) for 2.5 Hz, is compared on Fig. 12 with that computed by AMPFAC, normalized for equal response at  $\beta_S = 2\%$ . The pseudo ground-damping  $\beta_g$  was found to be 8.6%, in this case. Excellent agreement is found from  $\beta_S = 1.5\%$  to 15%, beyond which AMPFAC departs from the idealized straight line used by NBK, as might be expected. Since there is a finite maximum AFs/g of 6.61 at  $\beta_S = 0\%$ , the curve must move toward this value as  $\beta_S$  approaches zero.

The use of a decaying sinusoid to describe an earthquake is not unreasonable, as ground motion tends to a harmonic oscillation with increasing distance from the source (6). The profile of the San Francisco, California earthquake accelerogram of March 22, 1957, taken at Golden Gate Park (7), is very much like that of a decaying sinusoid, with a constant rate of damping. The decaying sinusoid also produces structure and equipment responses very similar to those resulting from random motion or an artificial earthquake (2,9)<sup>g</sup>.

A modified form of AMPFAC was tested using a decaying square wave, rather than a sinusoid, to represent the impulsive type of ground motion often ascribed to earthquakes and frequently used as the basis for earthquake time-histories. The results for  $\beta_S = 5\%$  and  $\beta_e = 2\%$  showed an AFe/s about 13% lower than for a decaying sine wave giving the same peak structure response.

A number of previous attempts have been made to produce equipment-motion spectra without resorting to time-history or random-motion analyses (8,9,10,11). Of these, Bigg's recent work (10) is the most comprehensive and probably the most accurate. A sample case was checked using the method described above with Biggs' method, assuming equal peak equipment-to-structure amplification factors. The combined response to a structure with four modes of vibration agreed within about 4%.

In general, responses determined by one of the referenced approximate methods are ultra conservative (12), while others show results which are more compatible with conventional methods (13) - see also Table 1. This is partly attributed to the exaggeration of the peak equipment amplification factors, where the highest values are used to cover all parts of the spectrum. The method described above attempts to keep amplification factors consistent with those found from more rigorous solutions.

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<sup>g</sup>See also comparisons of AF's in Table 1.

Response spectra were prepared for the El Centro, California 1940 (N-S) earthquake, using an adaptation of SPECEQ (5). Figures 13 and 14 for El Centro should be compared directly with Figures 8 and 10, as the cases are the same.

Fig. 13 shows similar equipment response beyond 33 Hz as was found in Fig. 8 using AMPFAC. The recommended design curve shown on Fig. 13 rolls off high-frequency response toward ground motion at 33 Hz, intersecting the default value equivalent to maximum structure motion at about 0.34g.

It is pointed out by Chen (14) that high-level equipment response well above 33 Hz (the 'whip' effect) is in doubt. Another computer run was performed with El Centro using higher damping values for structure and equipment; this whip effect was almost totally eliminated beyond 40 Hz. It is believed that the impulse interval of 0.01 seconds for the ground motion time-history used was too coarse for accurate results in the high-frequency region of the spectrum and that the recommended design curve is more reasonable.

Fig. 14, shows good agreement with Fig. 10, where the amplified equipment motion merges with that of the equipment acting as a structure resting on the ground (on the low-frequency side). Equipment motion falls to the default value corresponding to the maximum structure acceleration at 4 Hz (the resonant frequency of the structure) on the high frequency side.

Some of the equipment-to-structure amplification factors obtained with AMPFAC are compared with El Centro in Table 2.

#### CONCLUSIONS

It is concluded that the method presented in this paper is simple and convenient, as well as reliable. It permits rapid determination of earthquake motions in structures and attached light-weight equipment, without the need for costly computer programs or carefully-adjusted earthquake time histories. Any acceptable ground-response spectrum or a table of structure amplification factors can be used directly to determine equipment motion spectra. Actual equipment motions and responses can then be determined, using conventional modal analysis.

A modified form of this simplified method has been used effectively a number of times to develop equipment motion spectra. These have been applied to the aseismic design of Canadian nuclear power-plant components.

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TABLE 1

COMPARATIVE EQUIPMENT AMPLIFICATION FACTORS

<u>Earthquake Motion</u>	<u>Structure Damping (%)</u>	<u>Equipment Damping (%)</u>	<u>AF Equipment to Structure</u>	<u>Ref.</u>
Random Motion	2	0.5	20	2
Decaying Sine	2	0.5	23.9	AMPFAC
4 earthquakes	4	0.5	24.8 max	10
Decaying Sine	4	0.5	17.3	AMPFAC
Artificial E.Q.	5	0.5	15	9
Decaying Sine	5	0.5	15.7	AMPFAC
Simulated E.Q.	2	1	21.8 max	11
Decaying Sine	2	1	18.8	AMPFAC
Simulated E.Q.	5	0.5	20.2 max	12
Simulated E.Q.	5	0.5	+16.1	12
Decaying Sine	5	0.5	16.0	12

TABLE 2

EL CENTRO EARTHQUAKE VS. AMPFAC EQUIPMENT AMPLIFICATION FACTORS

Structure damping 5%  
Equipment damping 0.5%

<u>Frequency Range (Hz)</u>	<u>Amplification Factor AFe/s</u>			
	<u>El Centro</u>		<u>AMPFAC</u>	
0.08 - 0.4 (Displacement)	+7.73		12.7	
	5.86 avge			
0.4 - 3 (Velocity)	+13.8		13.9	
	10.7 avge			
3 - 10 (Acceleration)	+13.4		15.7	
	12.8 avge			
<u>Frequency</u>	<u>El Centro</u>		<u>AMPFAC</u>	
(Acceleration to ground)	<u>Design</u>	<u>Actual</u>	<u>Design</u>	<u>Actual</u>
10	13.80	*13.8	*13.1	13.1
15	5.88	* 5.88	*10.7	10.7
20	3.82	* 8.18	* 8.50	9.40
25	3.05	* 6.08	* 5.84	8.57
30	* 3.05	5.09	* 3.04	7.90
33	* 3.05	4.81	* 3.04	7.60

Values marked + are maximum-average (weighted)  
Values marked \* show best agreement

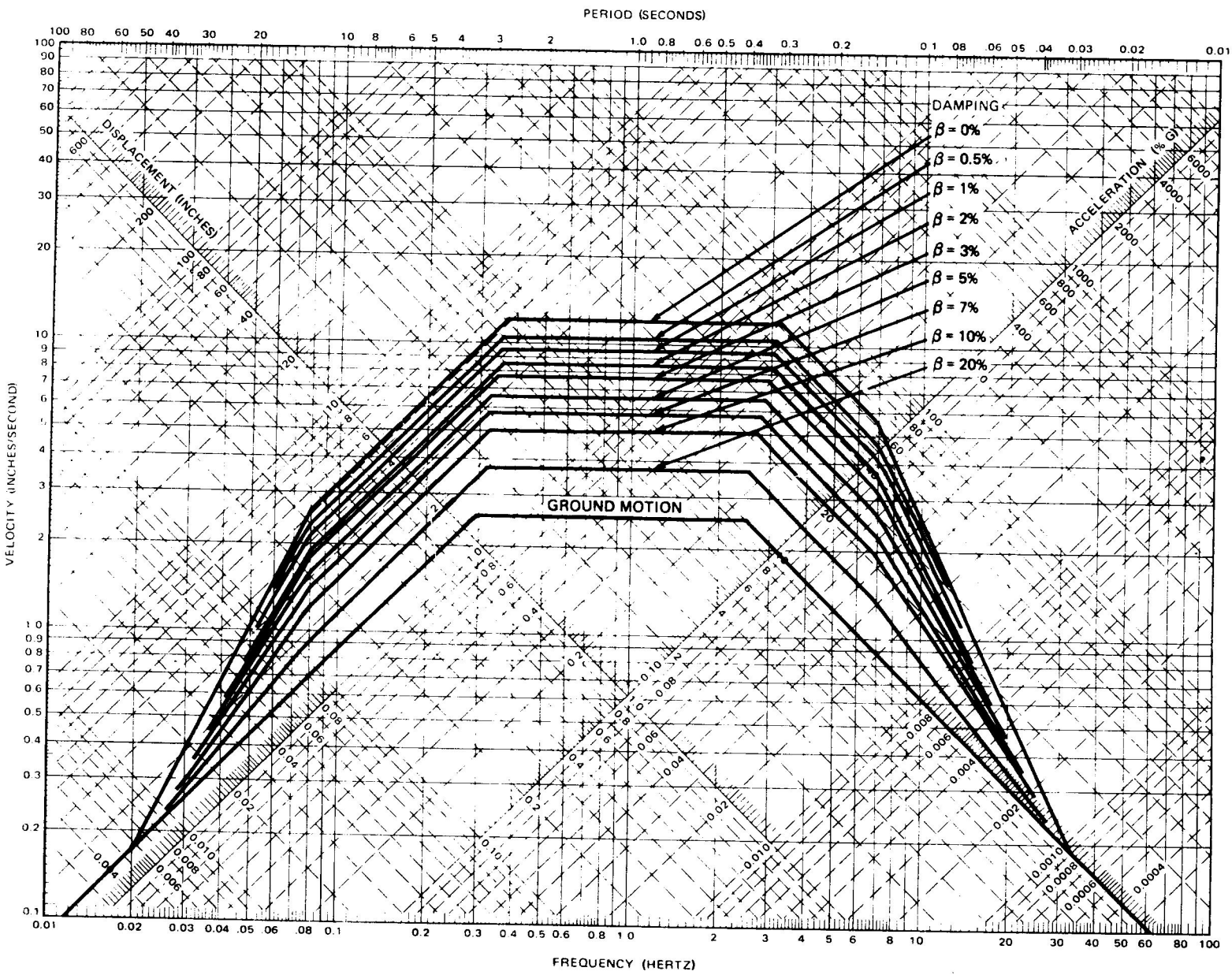
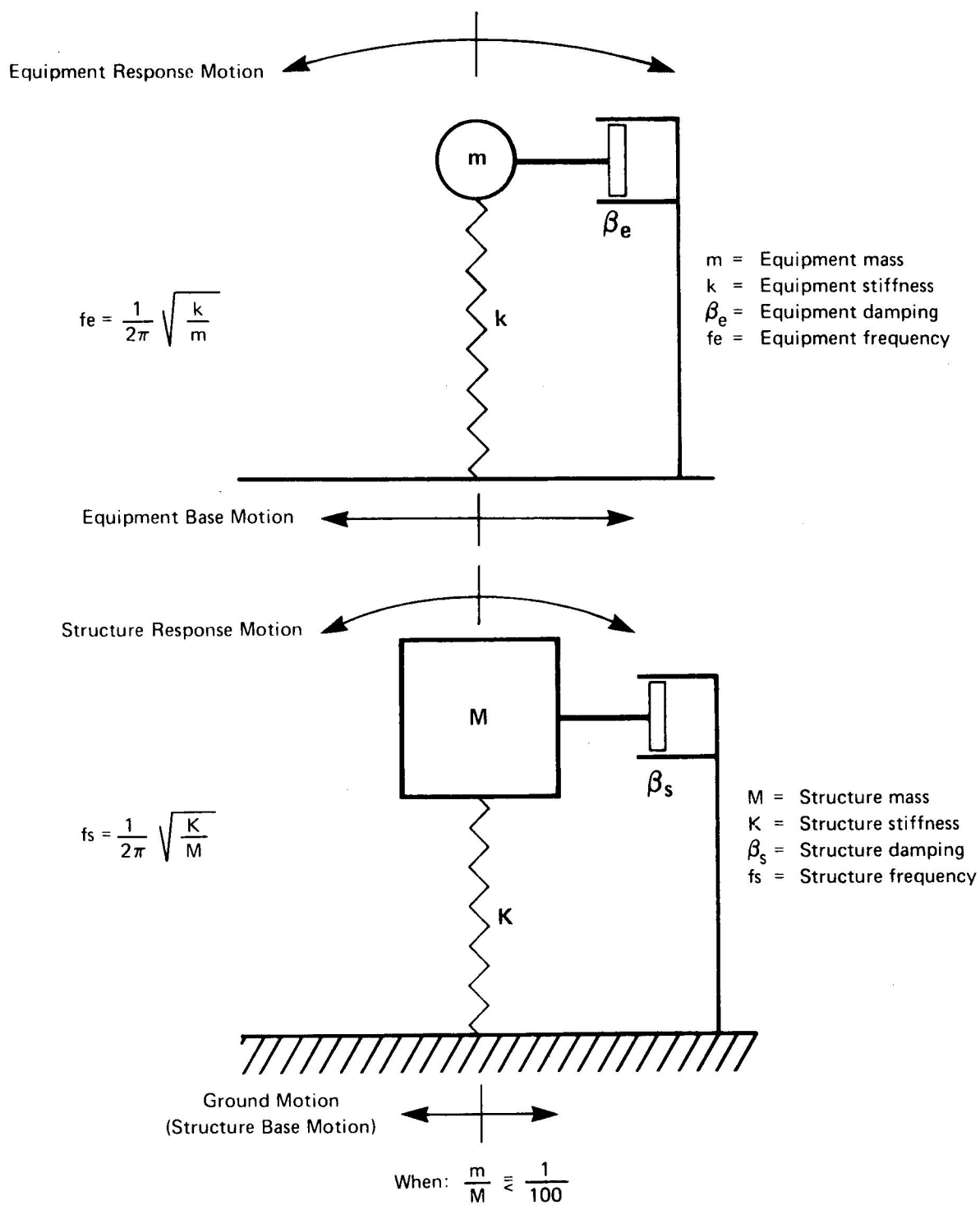


FIGURE 1 VELOCITY-NORMALIZED GROUND RESPONSE SPECTRA  
 --GROUND ACCELERATION 10% G



- Equipment is considered uncoupled from structure
- Equipment base motion = Structure response motion
- Resonance occurs when  $f_e = f_s$

FIGURE 2 TWO-LEVEL RESPONSE TO GROUND MOTION



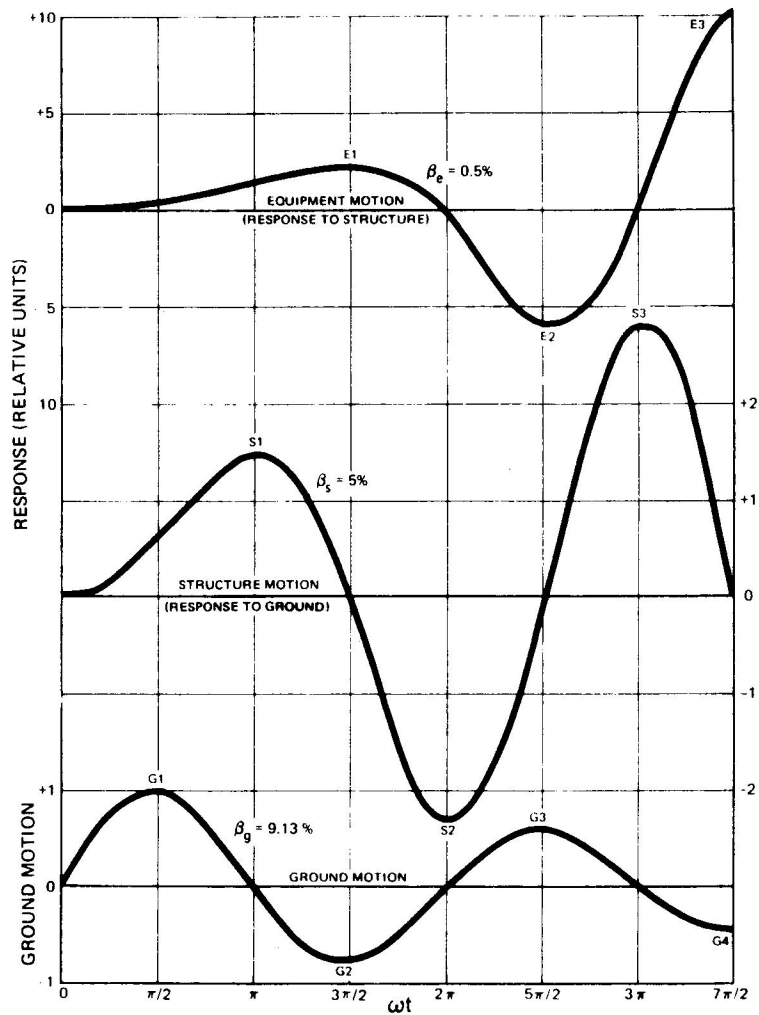


FIGURE 3 HARMONIC RESPONSE TO DECAYING SINUSOIDAL GROUND MOTION

91 45600 2  
FEB. 1975

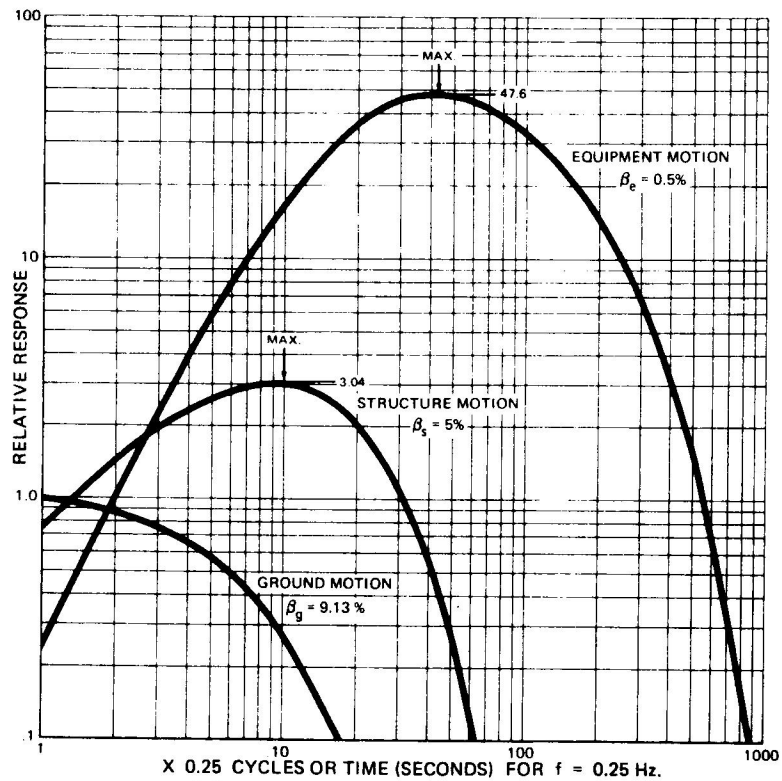


FIGURE 4 TIME-HISTORY OF RESPONSE

91 45600 4  
REV 1 FEB. 1975

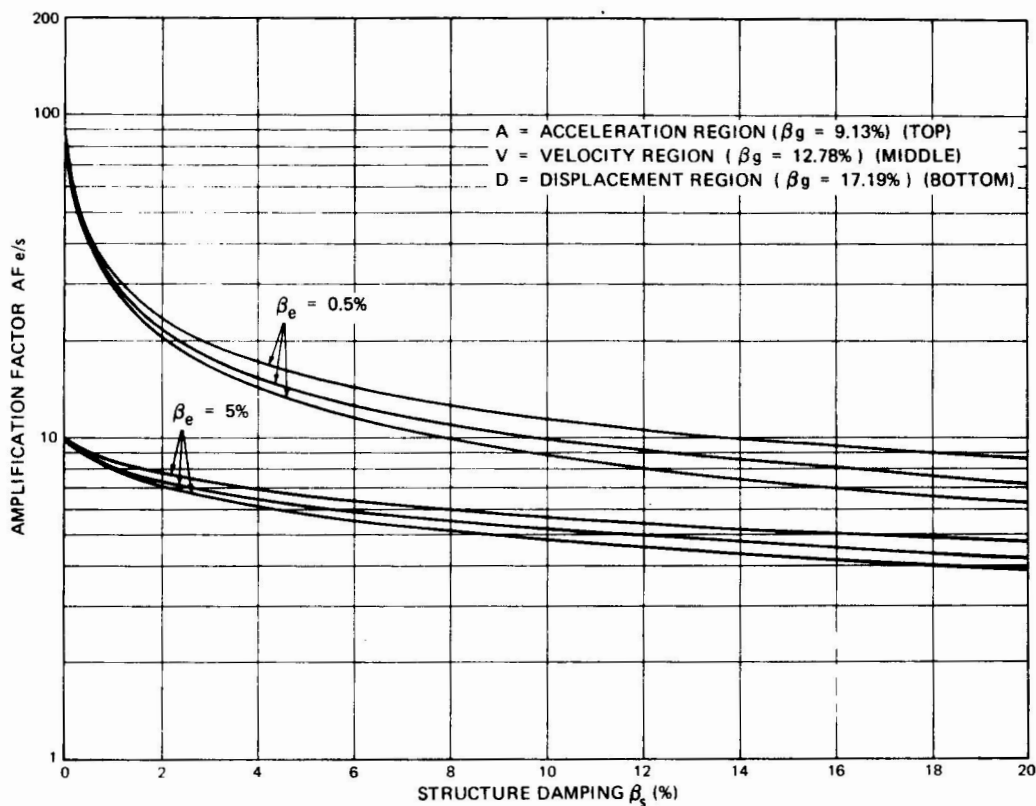


FIGURE 5 AMPLIFICATION FACTOR: EQUIPMENT - TO - STRUCTURE  $AF_e/s$  WITH VARIABLE PSEUDO GROUND DAMPING  $\beta_g$

91.45600 8  
 MAR 1975

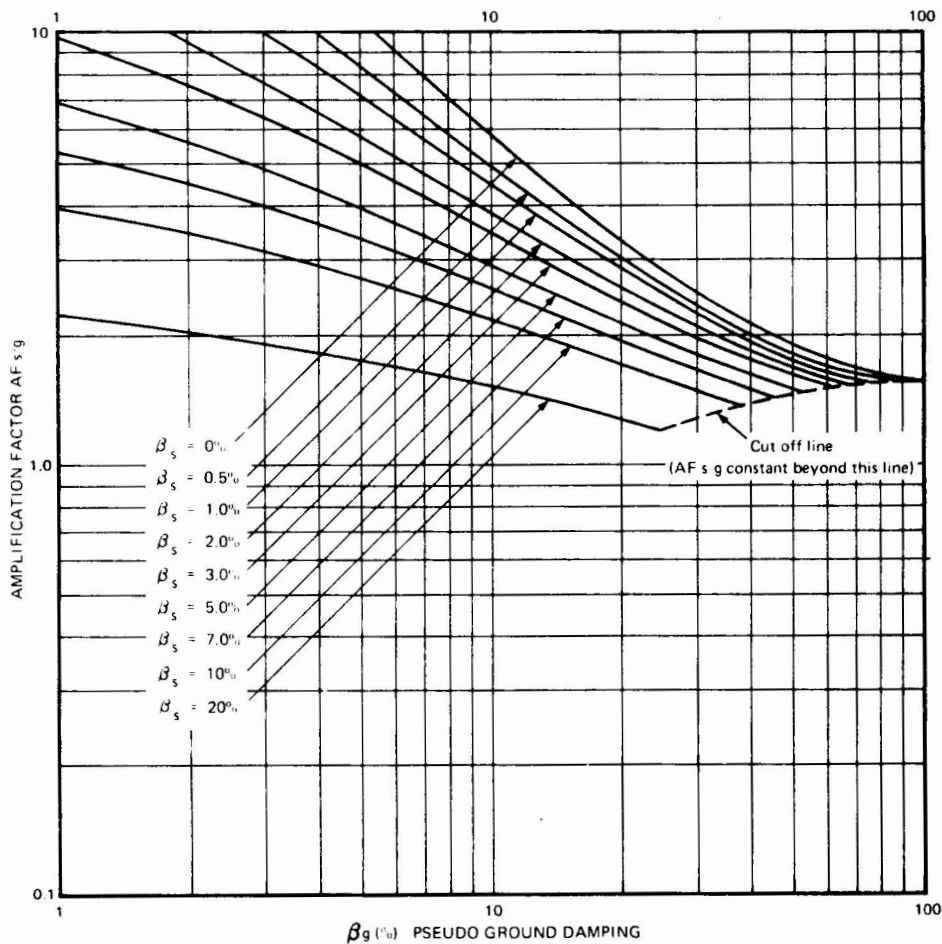


FIGURE 6 AMPLIFICATION FACTOR: STRUCTURE-TO-GROUND

91.45600 7  
 FEB. 1975

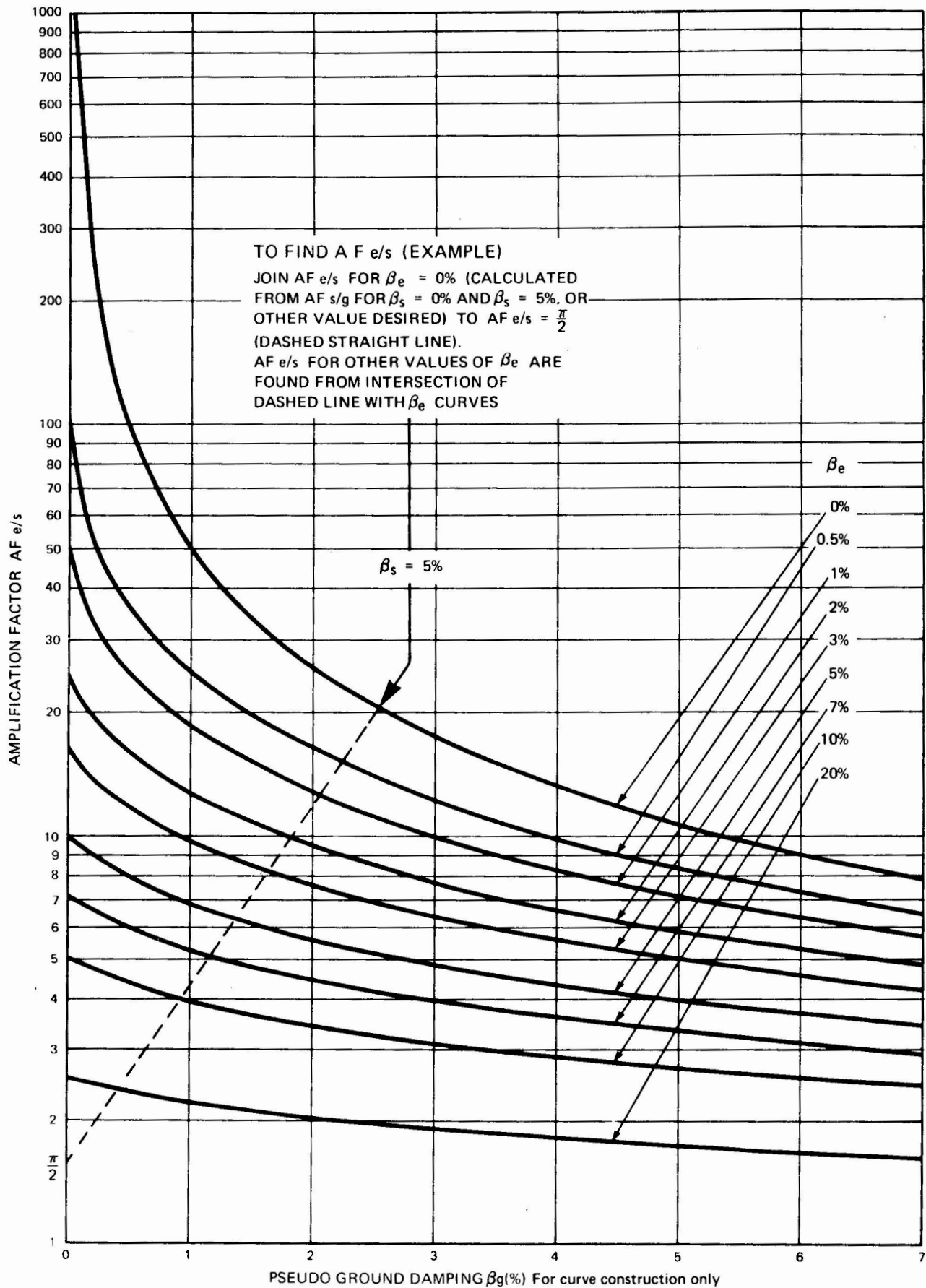


FIGURE 7 EQUIPMENT – TO – STRUCTURE AMPLIFICATION FACTOR

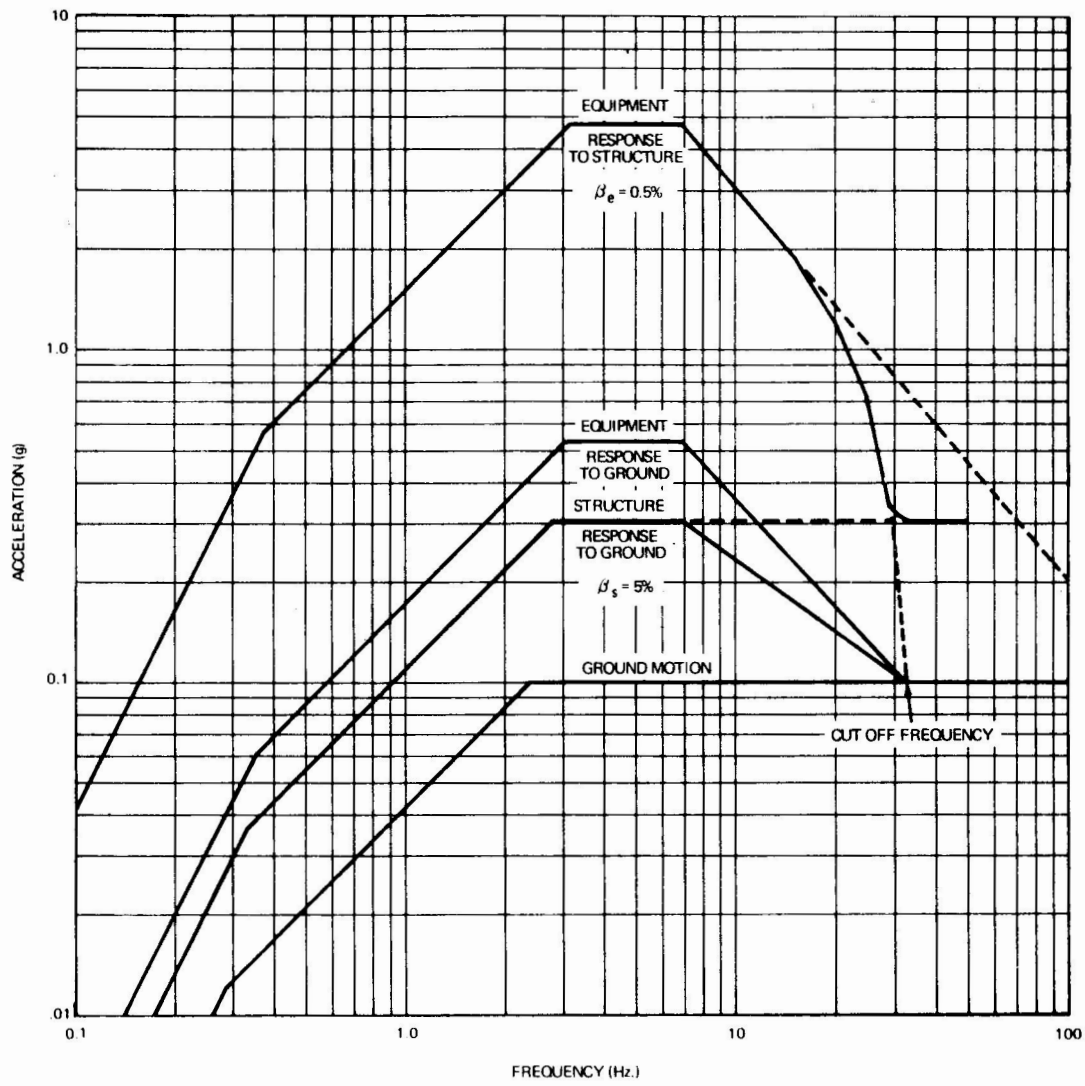


FIGURE 8 ENVELOPE RESPONSE SPECTRA

91 45600 10  
MAR. 1975

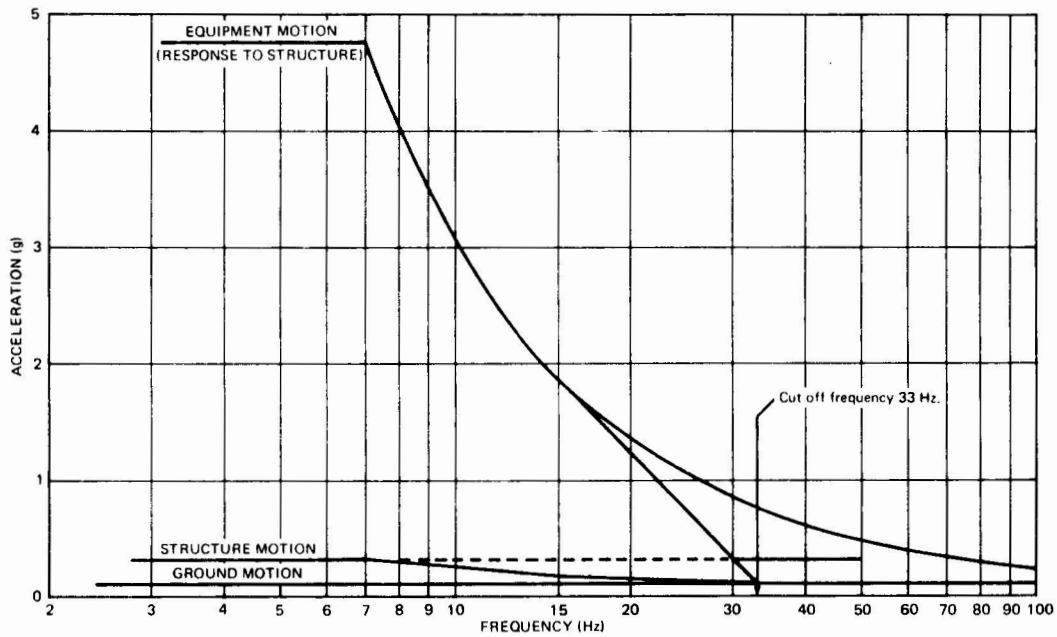


FIGURE 9 HIGH FREQUENCY RESPONSE OF EQUIPMENT (To cut-off)

91 45600 11  
MAR. 1975

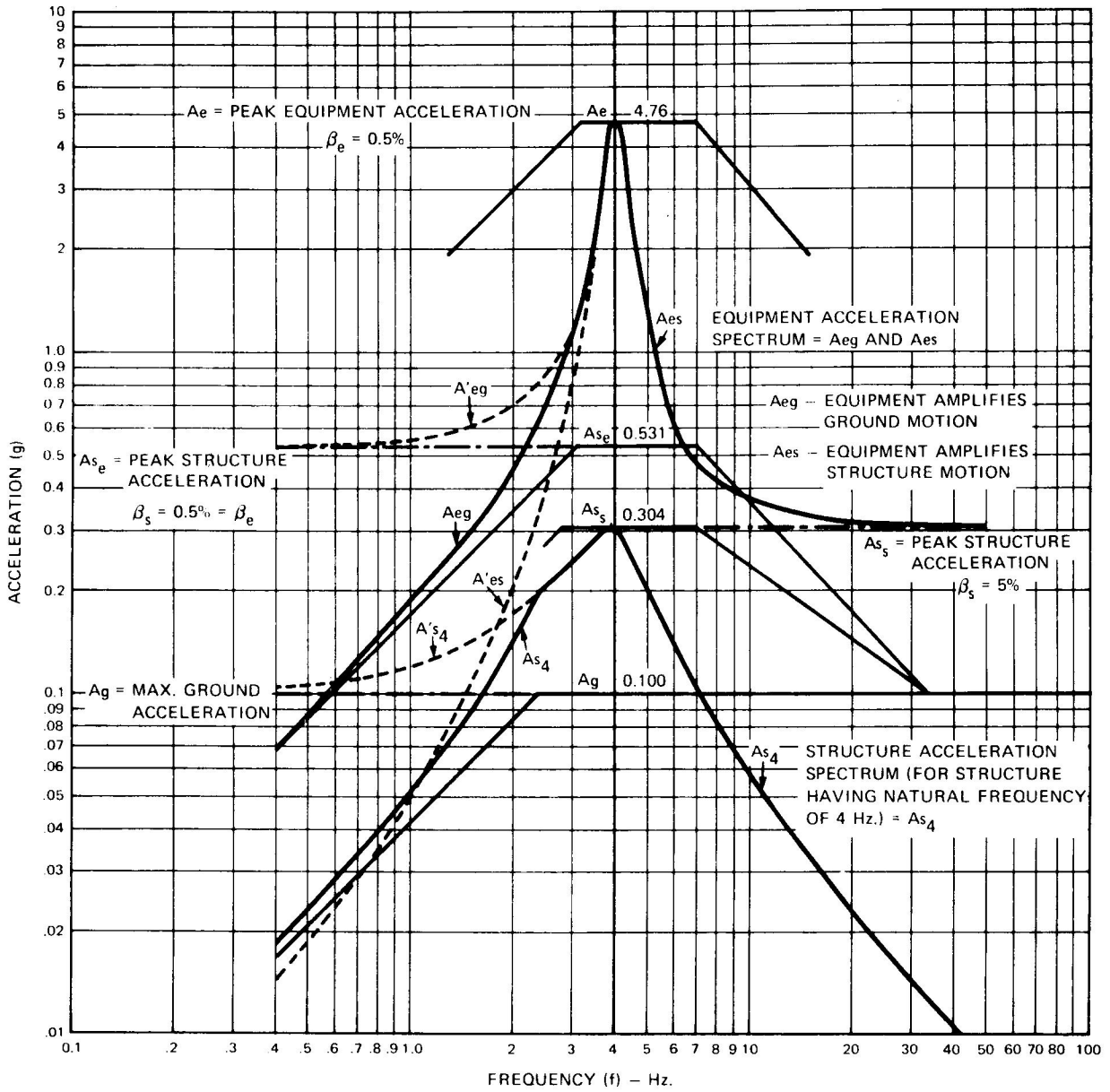


FIG. 10 AMPLIFIED MOTION SPECTRA ( $f_s = 4 \text{ Hz.}$ )

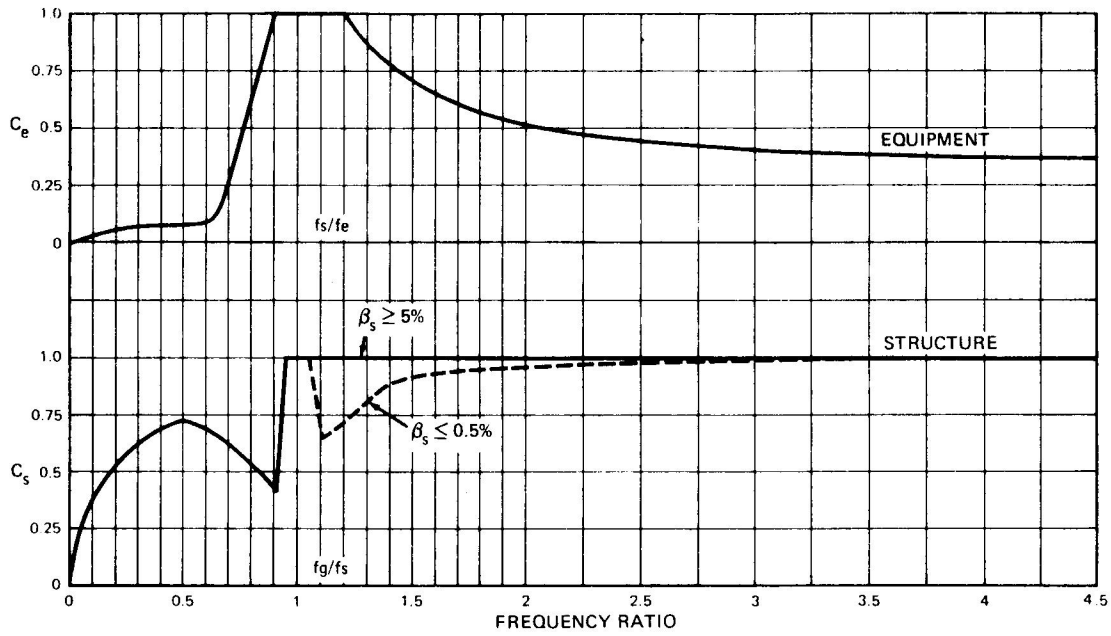


FIGURE 11 TRANSMISSIBILITY COEFFICIENTS

91 45600 13  
MAR 1975

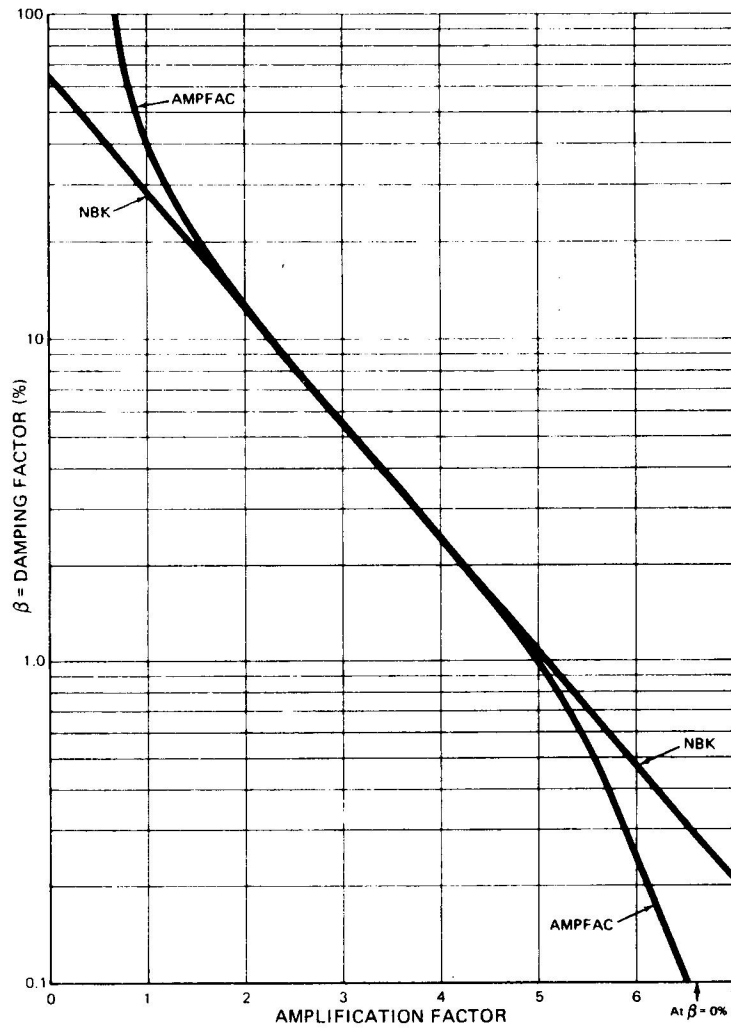


FIGURE 12 AMPLIFICATION FACTOR COMPARISON

91 45600 3  
REV. 1 FEB. 1975

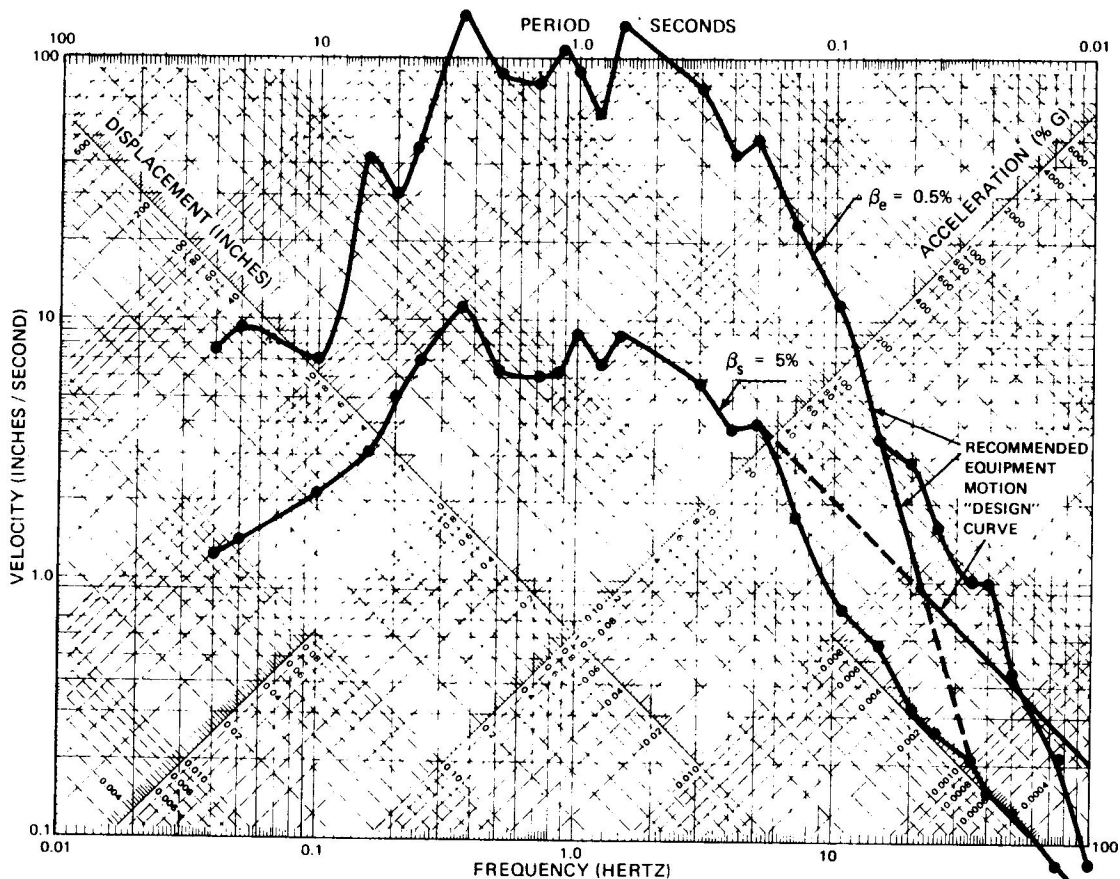


FIGURE 13 STRUCTURE AND EQUIPMENT ENVELOPE SPECTRA FOR EL CENTRO CALIF. 1940 (N-S) EARTHQUAKE (SCALED TO 0.1g)

91-45600 14  
MAR 1975

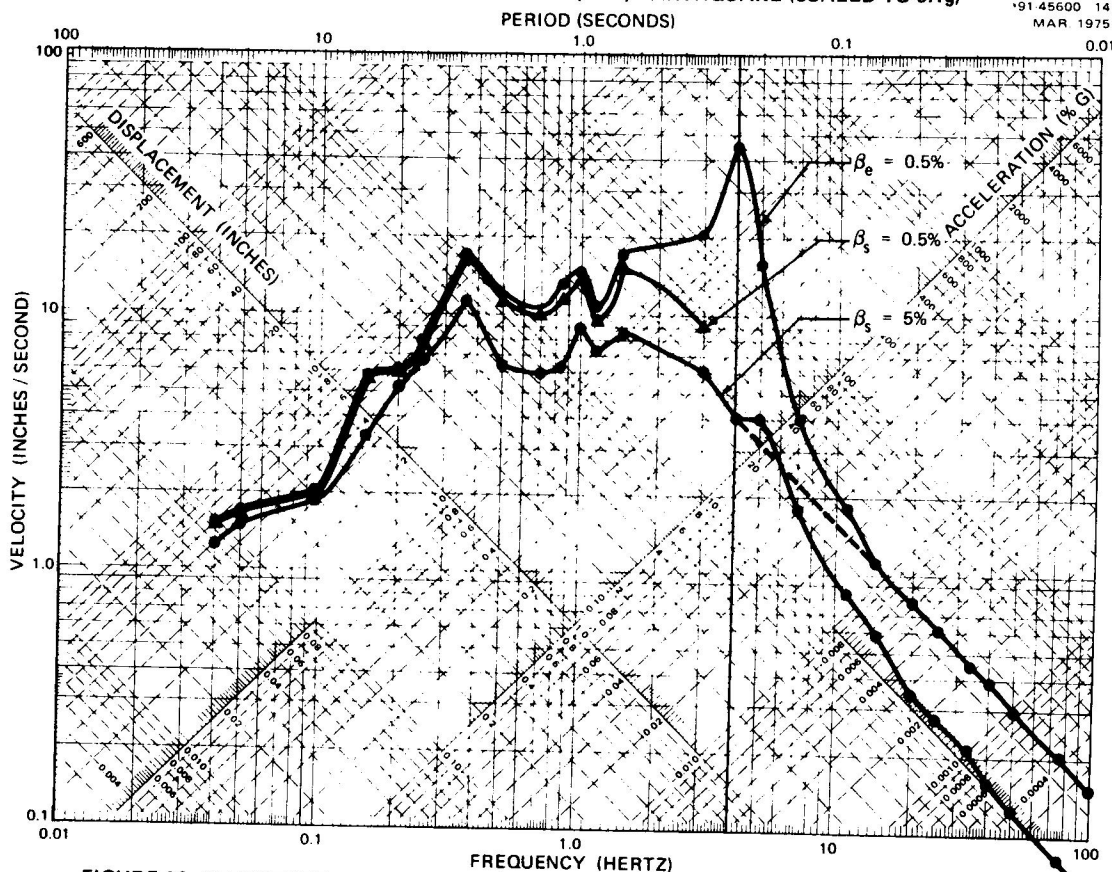


FIGURE 14 STRUCTURE MOTION SPECTRA AND EQUIPMENT MOTION SPECTRUM (AT 4 Hz.) FOR EL CENTRO CALIF. 1940 (N-S) EARTHQUAKE (SCALED TO 0.1g)

91-45600 15  
MAR 1975